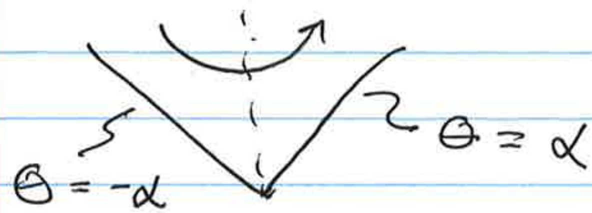
Moffat Eddies



swirling flow in a corner w/ angle 2α

Now $u_\theta = \frac{-\partial \psi}{\partial r}$, $u_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$

We wish to look at swirling flow

s.t. u_θ is sym. in θ, u_r is antisym in θ

$\therefore$ ψ is symmetric in θ!

We seek the general separable solution

$$\psi = \sum A_n r^{\lambda_n} f_{\lambda_n}(\theta)$$

where $f_{\lambda_n} = A_n \cos \lambda_n \theta + C_n \cos[(\lambda_n - 2)\theta]$

(note: this is the symmetric part - the antisym part would be sines)

Nearly always BC's force $\lambda = 0, 1, 2$. Here we look at a different case!

We have the conditions that $\psi\Big|_{\pm\alpha} = 0$

and, since $u_r\Big|_{\theta=\pm\alpha} = 0$, $\dfrac{\partial\psi}{\partial\theta}\Big|_{\theta\pm\alpha} = 0$

$\therefore\ f(\pm\alpha) = f'(\pm\alpha) = 0$

As $r \to 0$ only one term will dominate

(e.g., $r^{\lambda_1} > r^{\lambda_2} > r^{\lambda_3} \ldots$ as $r \to 0$)

since the real part of λ's are increasing

Just look at λ_1! $\psi \sim A_1 r^{\lambda_1} f_{\lambda_1}(\theta)$

Apply BC's:

$$A_1 \cos\lambda_1\alpha + C_1 \cos(\lambda_1 - 2)\alpha = 0$$

$$A_1 \lambda_1 \sin\lambda_1\alpha + C_1 (\lambda_1 - 2)\sin(\lambda_1 - 2)\alpha = 0$$

To have a solution, the determinant of the matrix must be zero:

$$\begin{vmatrix} \cos\lambda_1\alpha & \cos(\lambda_1 - 2)\alpha \\ \lambda_1 \sin\lambda_1\alpha & (\lambda_1 - 2)\sin(\lambda_1 - 2)\alpha \end{vmatrix} = 0$$

For this problem:

$$(\lambda_1 - 2)\sin(\lambda_1 - 2)\alpha \cos\lambda_1\alpha - \cos(\lambda_1 - 2)\alpha \cdot \lambda_1 \sin\lambda_1\alpha = 0$$

We can rewrite this as

$$-(\lambda_1 - 1)\sin 2\alpha = \sin\left[2(\lambda_1 - 1)\alpha\right]$$

If $2\alpha > 146^\circ$ this has a real solution.

But if $2\alpha < 146^\circ$ the solution is complex!

Let $\lambda_1 - 1 = p + iq$

$$\therefore r^{\lambda_1} = r^{1+p+iq} = r^{1+p} r^{iq}$$

$$= r^{1+p} e^{iq\ln r} = r^{1+p}\left[\cos(q\ln r) + i\sin(q\ln r)\right]$$

As $r \to 0$ $\ln r \to -\infty$ $\therefore$ if $q \neq 0$ (e.g. $2\alpha < 146^\circ$)

then we get an infinite number of zeros!

Because $\psi = 0$ on the walls, if $\psi = 0$ at some r we would have a dividing streamline!

$\therefore$ an infinite # of closed cells going into the corner!